

Center of Mass Motions

When analyzing collisions between several objects, physicists define a point in space known as the *center of mass* of the system of objects in order to simplify the analysis of motions before and after collisions. In particular, the center of mass of a colliding system that experiences no net outside force is defined as a point that moves at a constant velocity before, during and after the collisions. In this assignment you will have an opportunity to predict the location of the center of mass of a system consisting of (a) two carts of equal mass, and (b) two carts of unequal mass as they undergo collisions. Finally you can test your predictions by determining how your predicted center of mass moves in each of the collisions.

1. Preliminary Questions

Note: You will receive **full credit for each prediction** made in this preliminary section whether or not it matches conclusions you reach in the next section. As part of the learning process it is important to compare your predictions with your results. **Do not change your predictions!**

- (a) Imagine that we have two identical carts on a track. Where would you expect the “mass center” of the two-cart system to be? In other words, at what percentage of the distance between the centers of the carts do you expect the system center of mass to be? Draw an ‘X’ on the diagram below. Explain the reasons for your choice.



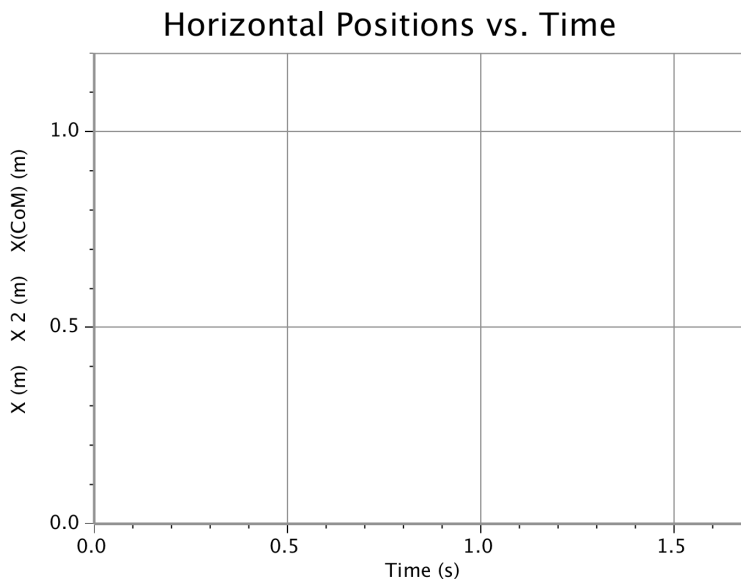
- (b) Now consider a system consisting of Cart 1 that has twice as much mass as Cart 2. Where would you expect the center of mass of this two-cart system to be? Draw an ‘X’ on the diagram below. At what percentage of the distance between the centers of the carts do you expect the system center of mass to be? Explain the reasons for your choice.



2. Activity-Based Questions

Center of Mass Motion in a 1D Collision for Equal Mass Carts: Before answering the questions below, open the Logger *Pro* experiment file <CoMMotions-EqualMass.cmb>. It has the movie <Equal_Mass_Colliding_Carts.mov> inserted. The movie, recorded at 10 frames/second, shows two identical carts moving on a track. The movie has been scaled to meters.

- (a) Click the **Add Point** icon ($\oplus$) once and then click in each frame of the movie on the yellow dot at the center of Cart 1 (the cart on the left) to create X:Y series data. Notice the x vs. t graph that appears. Sketch the data points in the figure below and label them as Cart 1. Rewind the movie (⏮) and use the **Set Active Point** tool (⏮) to select the X2:Y2 series. Click **Add Point** one time and click on the center of Cart 2 (the one on the right) in each frame. Sketch its graph in the figure that follows. Label it as Cart 2. Rewind the movie (⏮) again, use the **Set Active Point** tool (⏮) to select the X(CoM):Y(CoM) series. Then click a point where you expect the center of mass to be in each frame. (**Hint:** It might help to hold a ruler with millimeter markings up to the computer monitor.) Add these data to your sketch and label it properly.



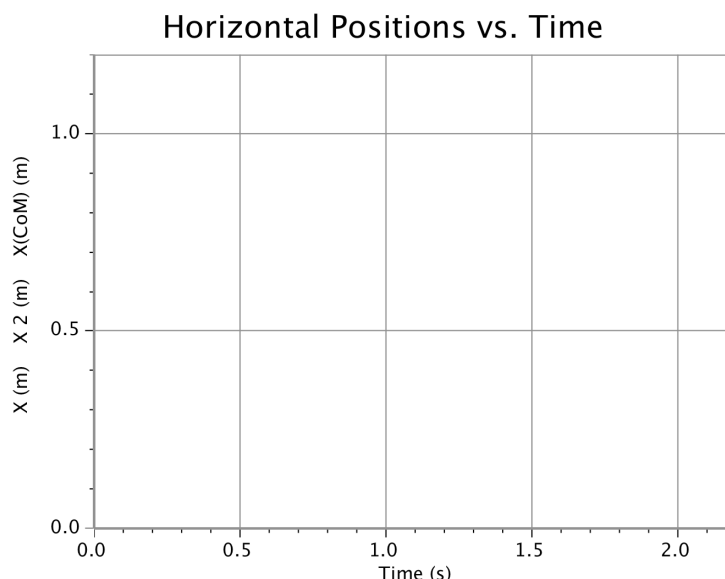
- (b) Explain how you estimated the center of mass of the system of two carts with equal mass.
- (c) How does the shape of the center-of-mass graph differ from the graphs for Cart 1 and Cart 2? Are the lines straight, jagged, curved, or some other shape?

Center of Mass Motion in a 1D Collision for Unequal Mass Carts: Before answering the questions below, open the Logger *Pro* experiment file <CoMMotions-UnEqualMass.cmb>. The movie <Unequal_Mass_Colliding_Carts.mov> has already been inserted. The movie, recorded at 10 frames/second, shows Cart 1 on the left with iron block attached to double its mass. The movie is scaled in meters.

- (d) Click the **Add Point** tool () once and then click in each frame of the movie on the yellow dot at the center of Cart 1 (the cart on the left) to create X:Y series data. Notice the x vs. t graph that appears. Sketch the data points in the figure below and label them as Cart 1.

Rewind the movie () and use the **Set Active Point** tool () to select the X2:Y2 series. Click **Add Point** one time and click on the center of Cart 2 (the one on the right) in each frame. Also sketch the data points in the figure below. Label it as Cart 2.

Rewind the movie () again, use the **Set Active Point** tool () to select the X(CoM):Y(CoM) series. Then click a point where you expect the center of mass to be in each frame. (**Hint:** It might help to hold a ruler with millimeter markings up to the computer monitor.) Add this data to your sketch and label it properly.



- (e) Explain how you estimated the center of mass of this unequal cart mass system.
- (f) How does the shape of the center-of-mass graph differ from the graphs for Cart 1 and Cart 2? Select the section of the center-of-mass graph *before* the collision and fit it to a straight line. Then select the section *after* the collision and fit it to a straight line. What are the slopes of the two fitted lines? Do the two lines have the same slope, or does the center-of-mass graph bend at the moment of collision?

(g) If you marked the centers of mass correctly, you should have gotten a very straight line for the center of mass graph. If slopes of the two fitted lines you got in part (f) differ by more than 5%, think about different ways of determining where the center of mass is until you get a straight-line graph. Then explain what you finally did to get a straight line.

(h) If the graph of the center of mass position versus time is a straight line, what can you say about the velocity of the center of mass?

3. Reflections on Your Findings

(a) Physicists define the center of mass of two objects (#1 and #2) with masses m_1 and m_2 at locations x_1 and x_2 to be

$$x_{CoM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad (\text{Eq.1})$$

The velocity of the center of mass is the derivative with respect to time of the x_{CoM} . Take the derivative of equation (1) with respect to time and write the result in the box below labeled Eq. (2).

Hints: x_1 and x_2 are functions of time. Also $v_{CoM} = dx_{CoM}/dt$ with $v_1 = dx_1/dt$ and $v_2 = dx_2/dt$.

(Eq. 2)

- (b) If we define the momentum of an object to be its mass multiplied by its velocity, then the x -component of the *total* momentum of the two-cart system would be sum of the momenta of the two carts:

$$p_{\text{system},x} = m_1 v_1 + m_2 v_2 \quad (\text{Eq. 3})$$

Compare equations (2) and (3). What can you conclude about how $p_{\text{system},x}$ is related to the center-of-mass velocity?

- (c) If the total momentum of a system is a constant, then we say that the total momentum is *conserved*. A consequence of Newton's laws of physics is that the total momentum of a system is conserved whenever the net external force on the system is zero. Describe all the external forces acting on this system and explain what the net external force is. Is it zero or approximately zero?